HEAT CONDUCTION WITH EXTERNAL COOLING OF GAS-TURBINE ROTOR BLADES

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With relevance to the study of external cooling of the working blades of gas turbines by air-liquid mixtures, a solution has been obtained for the problem of heat conduction of an infinite plate with periodic variations of the heat transfer coefficient and of the temperature of the surrounding medium.

Analysis of the causes which hinder the widespread use of well-known methods of internal cooling applied to the rotor blades of gas turbines leads one to consider how to achieve external cooling without interfering with rotor design.

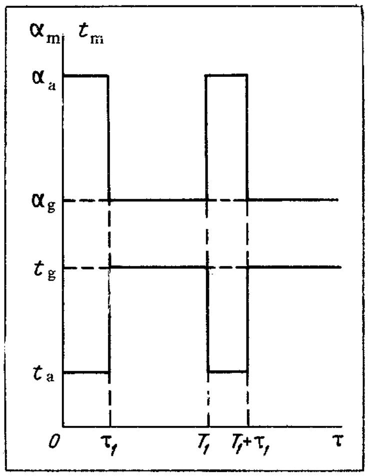

Fig. 1. The heat-transfer coefficient and the temperature of the surrounding medium as a function of time.

Reference [1] has suggested cooling of the blades of a turbine rotor by an air-liquid mixture discharged from a certain number of hollow stator blades. In this system drops of liquid falling onto the surface of the nozzle blades are heated and vaporized. If the temperature of the cooled blades is comparatively high (greater than 600° C) then, for a small number of cooling elements, the drops will be evaporated from the surface of the blades and the heat-transfer process between the medium and the blades will have a pronounced periodic character, due to the difference in the heat-transfer coefficients, and in the temperatures of the gas and the cooling agent.

If the variations of the heat-transfer coefficient and of the temperature of the medium are as shown in Fig. 1, then solution of the heat-conduction problem for an infinite plate would allow us to estimate the possibility of, and the special features of, external cooling of turbine blades.

Prikhod'ko [6], under the direction of the present author, has determined the temperature field of a plate for an arbitrary variation with time of the heat-transfer coefficient and the temperature of the surrounding medium. This was achieved by reducing the problem to a Sturm-Liouville system, equivalent to a Fredholm integral equation of the second kind, and by applying a special bilinear series to expand the kernel of the integral equation.

However, it is difficult to use this solution for the periodic laws of variation of medium temperature and heat-transfer coefficient that interest us, since t_m and α_m cannot be expressed in terms of analytical functions. Representation of these by the appropriate Fourier series leads to extreme mathematical difficulties in attempting to proceed from the above solution.

The most important aspects of our problem are the questions of what is the mean-integral plate temperature in a quasi-steady state, i.e., by how much can

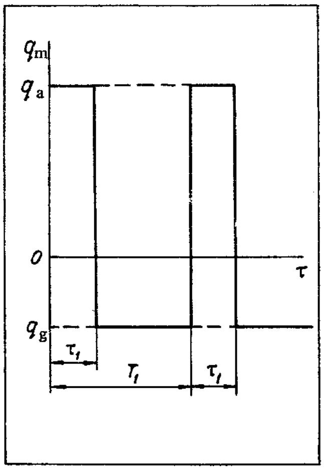

Fig. 2. Heat flux at the plate surface as a function of time.

the temperature of a hot plate, located in a medium with periodic variation of t_m and α_m, be reduced, and what will then be the range of temperature oscillations at its surface.

To answer the first question, we start from the thesis that for a quasi-steady state ($\tau \to \infty$) the mean-integral temperature of all points of the body does not depend on its dimensions when the action of the medium on the body is periodic. On this basis we solve the problem of determining $\bar{t}(\infty)$ for a body that is thin in the thermal sense. The solution for arbitrary periodic laws of variation of t_m and α_m has been given in [2]. If we use this solution, then, for the laws of variation of t_m and α_m considered, we obtain an expression for the body temperature at the end of the m-th period:

$$t(T_1)_m = t_0 + \exp\{-mk[\alpha_a\tau_1 + \alpha_g(T_1-\tau_1)]\} + \\ + \left(t_a[1-\exp(-k\alpha_a\tau_1)]\exp[-k\alpha_g(T_1-\tau_1)] + \\ + t_g\exp[-k\alpha_g(T_1-\tau_1)]\right)\left(1-\exp\{-k[\alpha_a\tau_1 + \\ + \alpha_g(T_1-\tau_1)]\}\right)^{-1} \times \\ \times \{1-\exp\{-mk[\alpha_a\tau_1 + \alpha_g(T_1-\tau_1)]\}\}. \quad (1)$$

Hence, for sufficiently large m, i.e., for a quasi-steady state,

$$t(T_1)_m = t(\infty) = \left(t_a[1-\exp(-k\alpha_a\tau_1)]\exp[-k\alpha_g \times \\ \times (T_1-\tau_1)] + t_g\exp[-k\alpha_g(T_1-\tau_1)]\right) \times \\ \times \left(1-\exp\{-k[\alpha_a\tau_1 + \alpha_g(T_1-\tau_1)]\}\right)^{-1}. \quad (2)$$

The temperature τ_1 of a thin body within the first section, and in the second section $T_1 - \tau_1$, of the (m + 1)-th period (for sufficiently large m) are calculated to be

$$t(T_1)_{m+1,\tau_1} = t(\infty)_{\tau_1} = t_a - [t_a - t(T_1)_m]\exp(-k\alpha_a\tau), \quad (3)$$

$$t(T_1)_{m+1,T_1} = t(\infty)_{T_1} = \\ = t_g - (t_a - t_g)\exp[-k\alpha_g(T_1-\tau_1)] - \\ - [t_a - t(T_1)_m]\exp\{-k[\alpha_a\tau_1 + \alpha_g(\tau-\tau_1)]\}. \quad (4)$$

If we substitute the value of $t(T_1)_m$ from (2) into (3) and (4), and determine the mean-integral temperature of the thin body in the quasi-steady state over the period T_1, the final expression is obtained in the form

$$\bar{t}(\infty) = \frac{\tau_1}{T_1}t_a + \frac{T_1-\tau_1}{T_1}t_g - \left(\frac{1}{\alpha_g} - \frac{1}{\alpha_a}\right)\frac{t_g - t_a}{kT_1} \times \\ \times \frac{[1-\exp(-k\alpha_a\tau_1)]\{1-\exp[-k\alpha_g(T_1-\tau_1)]\}}{1-\exp\{-k[\alpha_a\tau_1 + \alpha_g(T_1-\tau_1)]}. \quad (5)$$

The last term of (5) expresses the influence of the periodic law of variation of α_m on the mean-integral body temperature. For a high oscillation frequency of α_m and t_m, expression (5) takes the simpler form

$$\bar{t}(\infty) = \frac{t_a\alpha_a\tau_1 + t_g\alpha_g(T_1-\tau_1)}{\alpha_a\tau_1 + \alpha_g(T_1-\tau_1)}, \quad (6)$$

since for x < 0.01, to a sufficient degree of accuracy,

$$\exp(-x) = \frac{1}{1+x}.$$

By introducing the equivalent heat-transfer coefficient

$$\alpha_{eq} = \alpha_a\frac{\tau_1}{T_1} + \alpha_g\frac{T_1-\tau_1}{T_1}, \quad (7)$$

we can represent the solution of (1) in the form

$$t(T_1)_m = t(T_1 m) = t(\tau) = \\ = A\exp\left(-\frac{k}{T_1^2}\alpha_{eq}\tau\right), \quad (8)$$

where $A = f(t_0, t_a, t_g, \alpha_a, \alpha_g, k, T_1, \tau_1)$, and does not depend on τ. Therefore the temperature of a thin body for periodic variation of α_m and t_m proves to be an exponential function of time, as measured by the integers T_1. This fact allows us to determine experimentally the equivalent heat transfer coefficient by the methods of regular regime theory, and not only for thin bodies. In the case considered, for high oscillation tion frequencies of α_m and t_m, the temperature oscillations at all points of the body, as was shown above, are small in comparison with the range of oscillations of t_m, and so, by neglecting these oscillations, we can assume that the temperature field of the body will vary as it would if the body were acted on by a medium with constant temperature $t_m = \bar{t}(\infty)$ at a constant heat transfer coefficient $\alpha = \alpha_{eq}$.

Thus, if α_g, α_{eq}, T_1, and τ_1 are known, we can calculate the mean temperature $\bar{t}(\infty)$ of the plate, and estimate, in first approximation, the effectiveness of external cooling of the rotor blades in gas turbines.

Having determined the mean-integral temperature for a quasi-steady state, we can explain how the plate temperature oscillates relative to its mean value [5]. To do this we solve the following problem.

We are given an infinite plate of thickness 2R at a temperature of $t(x, 0) = \bar{t}(\infty)$. Both surfaces of the plate are exposed to a periodically varying heat flux $q_m = \alpha_m[\bar{t}(\infty) - t_m]$ (Fig. 2). We have to find the temperature distribution through the plate some instant of time.

We have

$$\frac{\partial t(x,\tau)}{\partial\tau} = a\frac{\partial^2 t(x,\tau)}{\partial x^2}, \quad (9)$$

$$t(x, 0) = \bar{t}(\infty) = t_0, \quad (10)$$

$$\lambda\frac{\partial t(R,\tau)}{\partial x} + q_m = 0, \quad (11)$$

where

$$q_m = \begin{cases} q_a & \text{for } 0 < \tau < \tau_1, \\ q_g & \text{for } \tau_1 < \tau < T_1; \end{cases} \quad (12)$$

$$\frac{\partial t(0,\tau)}{\partial x} = 0. \quad (13)$$

We solve the problem by an operational method.

As is known [3], the solution of the heat-conduction equation for the transform of a function, taking into account (13), has the form

$$\Phi(x, s) - \frac{t_0}{s} = c_1 \mathrm{ch}\sqrt{\frac{s}{a}}\,x. \quad (14)$$

The transform of the heat flux is

$$Q(s) = L[q_m] = \frac{q_a}{(T_1-\tau_1)s},$$

$$\frac{T_1[1-\exp(-\tau_1 s)]-\tau_1[1-\exp(-T_1 s)]}{1-\exp(-T_1 s)} =$$

$$= \frac{\Delta q}{s} = \frac{1-\exp(-\tau_1 s)-\frac{\tau_1}{T_1}[1-\exp(-T_1 s)]}{1-\exp(-T_1 s)}, \quad (15)$$

where

$$\Delta q = q_a - q_g \quad \text{and} \quad q_a \tau_1 = q_g(T_1-\tau_1).$$

If we substitute (15) into the boundary condition (11) written for the transform of the function, and taking into account (14), we find the solution for the transform

$$\Phi(x, s) - \frac{t_0}{s} = \frac{\Delta q}{\lambda s} \times$$

$$\times \frac{1-\exp(-\tau_1 s)-\frac{\tau_1}{T_1}[1-\exp(-T_1 s)]}{1-\exp(-T_1 s)} \times$$

$$\times \frac{\operatorname{ch}\sqrt{\frac{s}{a}}\,x}{\sqrt{\frac{s}{a}}\operatorname{sh}\sqrt{\frac{s}{a}}\,R}. \quad (16)$$

In order to go back to the original function we must apply the inverse Laplace transform and the Cauchy residue theorem. Here we can restrict the determination of residues to the poles located along the imaginary axis, i.e., in the roots of the equation $1-\exp(-sT_1) = 0$ (whence $s = \pm 2k\pi i/T$, where $k = 1$ 2, 3, ...), since the periodic part of the solution is determined only by

$$\sum \operatorname{res} = \sum_{\substack{k=-\infty \\ k\neq 0}}^{k=\infty} \Bigg[\frac{\exp(\tau, s)}{s} \times$$

$$\times \frac{1-\exp(\tau, s)-\frac{\tau_1}{T_1}[1-\exp(-T_1 s)]}{\frac{d}{ds}[1-\exp(-sT_1)]} \times$$

$$\times \frac{\operatorname{ch}\sqrt{\frac{s}{a}}\,x}{\sqrt{\frac{s}{a}}\operatorname{sh}\sqrt{\frac{s}{a}}\,R} \Bigg]_{s=\frac{2k\pi i}{T_1}} =$$

$$= \sum_{k=1}^{\infty} \left\{ \frac{\exp\left(\frac{2k\pi i}{T_1}\right)}{2k\pi i} \left[1-\exp\left(-\frac{2k\pi i}{T_1}\tau_1\right)\right] \times \right.$$

$$\times \frac{\operatorname{ch}\sqrt{\frac{2k\pi i}{aT_1}}\,x}{\sqrt{\frac{2k\pi i}{aT_1}}\operatorname{sh}\sqrt{\frac{2k\pi i}{aT_1}}\,R} - \frac{\exp\left(-\frac{2k\pi i}{T_1}\right)}{2k\pi i} \times$$

$$\left. \times \left[1-\exp\left(\frac{2k\pi i}{T_1}\tau_1\right)\right] \frac{\operatorname{ch}\sqrt{\frac{2k\pi i}{aT_1}}\,x}{\sqrt{-\frac{2k\pi i}{aT_1}}\operatorname{sh}\sqrt{\frac{2k\pi i}{aT_1}}\,R} \right\} =$$

$$= \sum_{k=1}^{\infty} \frac{A(\omega-\varphi)-B(\omega+\varphi)}{2k\pi\sqrt{\frac{k\pi}{aT_1}}(\operatorname{sh}^2 r\cos^2 r+\operatorname{ch}^2 r\sin^2 r)}, \quad (17)$$

where

$$A = \operatorname{ch} p\cos p\operatorname{sh} r\cos r + \operatorname{sh} p\sin p\operatorname{ch} r\sin r;$$

$$B = \operatorname{ch} p\cos p\operatorname{ch} r\sin r + \operatorname{sh} p\sin p\operatorname{sh} r\cos r;$$

$$\omega = \sin b\tau - \sin b(\tau-\tau_1);$$

$$\varphi = \cos b\tau - \cos b(\tau-\tau_1);$$

$$p = x\sqrt{\frac{k\pi}{aT_1}};\quad r = R\sqrt{\frac{k\pi}{aT_1}};\quad b = \frac{2k\pi}{T_1}. \quad (18)$$

Thus, the periodic component of the plate temperature is

$$t(x, \tau) - t_0 = \frac{\Delta q}{2\pi\sqrt{\frac{\pi}{aT_1}}\lambda} \times$$

$$\times \sum_{k=1}^{\infty} \frac{A(\omega-\varphi)-B(\omega+\varphi)}{(\operatorname{sh}^2 r\cos^2 r+\operatorname{ch}^2 r\sin^2 r)k^{3/2}}. \quad (19)$$

Since the remaining poles of function (16) lie at zero and on the left side of the real axis, the residues corresponding to these allow us to obtain the constant and aperiodic parts of the solution. However, there is no need to determine these, since the constant part of the solution, representing the mean deviation of plate temperature from the mean-integral value for a quasi-steady state, is negligibly small. The sign of this quantity depends on the sign of q_m at time zero. The sum of the series of exponentials, determining the aperiodic part of the solution, is important only for small τ, as reckoned from the start of examination of the process, and for $\tau \to \infty$ this sum tends to zero.

The largest temperature oscillations, of course, will occur at the plate surface.

For,

$$x = R,\quad p = r,\quad A = \frac{1}{2}\operatorname{sh} 2r,$$

$$B = \frac{1}{2}\sin 2r(\operatorname{ch}^2 r - \operatorname{sh}^2 r)$$

and

$$t(R, \tau) - t_0 = \frac{\Delta q}{2\pi\sqrt{\frac{\pi}{aT_1}}\lambda} \times$$

$$\times \sum_{k=1}^{\infty} \frac{\operatorname{sh} 2r(\omega-\varphi)-\sin 2r(\operatorname{ch}^2 r-\operatorname{sh}^2 r)(\omega+r)}{2(\operatorname{sh}^2 r\cos^2 r+\operatorname{ch}^2 r\sin^2 r)k^{3/2}}. \quad (20)$$

If we assume that for gas turbines with n = 50 rps, and with cooling agents supplied only from two blades of the nozzle diaphragm $r > 5(k)^{1/2}$, with $\operatorname{sh} r = \operatorname{ch} r$ in practice, we obtain

$$t(R, \tau) - t_0 = \frac{2 \sin \frac{\pi}{4} \Delta q}{\pi \sqrt{\frac{\pi}{aT_1}} \lambda} \times$$

$$\times \sum_{k=1}^{\infty} \frac{\sin \frac{k \pi \tau_1}{T_1} \cos \left[\frac{\pi}{4} - \frac{2k\pi}{T_1} \left(\tau - \frac{\tau_1}{2} \right) \right]}{k^{3/2}}. \quad (21)$$

The range of temperature oscillations of the plate surface is

$$\Delta t(R) = t(R, \tau_1) - t(R, 0) = \frac{\Delta q}{\pi \sqrt{\frac{\pi}{aT_1}} \lambda} \times$$

$$\times \sum_{k=1}^{\infty} \frac{\sin^2 k \pi \frac{\tau_1}{T_1}}{k^{3/2}} < \frac{\Delta q}{\pi \sqrt{\frac{\pi}{aT_1}}} \sum_{k=1}^{\infty} k^{-\frac{3}{2}} =$$

$$= \Delta q \frac{\sqrt{aT_1}}{2.133\lambda}, \quad (22)$$

since the sum $\sum_{k=1}^{\infty} k^{-\frac{3}{2}} = 2.612$ [7].

Putting $a = 6 \cdot 10^{-6}$ m^2/sec, $\lambda = 20$ W/m·deg and $T_1 = 2 \cdot 10^{-4}$ sec, we obtain

$$\Delta t(R) < 0.8 \cdot 10^{-6} \Delta q.$$

Cases are possible in which T_1 will have an appreciable value. Thus, for n = 50 rps, and with two spray nozzles, $T_1 = 0.01$ sec. In this case

$$\Delta t(R) < 5.6 \cdot 10^{-6} \Delta q,$$

and if $\Delta q = 10^6$ W/m^2, then $\Delta t(R) < 5.6°$, while if $\Delta q = 10^7$ W/m^2, we have $\Delta t(R) < 56°$ C.

The results of experiments have shown in [1, 4], that for values of the parameters that are typical of contemporary gas turbines, the value of Δq with external air-water cooling is the order of 10^6 W/m^2. Then $\Delta t(R) < 5.6°$. Subsequently, if it turns out in that $\Delta q = 10^7$ W/m^2, then even with unfavorable conditions (low cycle frequency) the range of oscillations of the blade surface temperatures $\Delta t(R)$ will be less than 56° C. On this evidence it can be asserted that possible oscillations of surface temperature of the rotor blades of a gas turbine with air-liquid cooling cannot be an obstacle to the practical application of this method of cooling.

NOTATION

Here t_m is the temperature of the medium; α_m is the heat transfer coefficient; T_1 is the period of oscillation of t_m, α_m; τ_1 is the time of action of the cooling agent on the body; t_g and t_a are the temperatures of the gas and of the cooling agent; α_a is the coefficient of heat transfer between the cooling agent and the body; α_g is the coefficient of the heat transfer between the gas and the body; $k = \psi F/c\gamma V$; ψ is the coefficient characterizing the nonuniformity of temperature distribution in the body; F and V are the surface area and volume of the body; γ and c are the density and specific heat of the body; a is the thermal diffusivity; q_m is the heat flux through the body surface; R is the half-thickness of the plate; λ is the thermal conductivity.

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